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SHORTER COMMUNICATIONS

A SOLUTION PROCEDURE FOR HEAT AND MASS TRANSFER IN MULTICOMPONENT LAMINAR JETS

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NOMENCLATURE

c_i,	mass fraction of species i;
D_{ij},	diffusion coefficient of species i;
D_{iT},	thermal-diffusion coefficient of species i;
D_{Ti},	diffusion-thermo coefficient of species i;
h,	specific enthalpy of mixture;
h_i,	specific enthalpy of species i;
I^*,	nondimensional enthalpy;
J_{iy},	radial mass flux species i;
J_{qy},	radial energy flux;
k,	thermal conductivity;
m_i,	molecular weight of species i;
n,	number of species;
$\tilde{P}r_i$,	Prandtl–Schmidt number;
T,	temperature;
u, u^*,	axial velocity (dimensional, nondimensional);
v,	radial velocity;
x,	axial coordinate;
y,	radial coordinate.

Greek symbols

α_{ij},	transformation parameter;
β_i,	transformation parameter;
γ_{ij}, γ_i,	reverse transformation parameters;
ζ_i,	'energy-species' variable;
ρ,	density;
μ,	viscosity.

INTRODUCTION

IN A MULTICOMPONENT flow system in which temperature gradients are present, lighter species tend to migrate towards higher temperatures. This phenomenon, known as the thermal-diffusion or Soret effect, has been observed experimentally [1] and found theoretically [2–5]. Similarly, diffusion-thermo or the Dufour effect is the generation of an additional heat flux due to concentration gradients. This, too, has been noted experimentally [6] and theoretically [7]. In multicomponent systems there is also an additional mass flux for each component resulting from the concentration gradients of all the other components present. All these effects comprise what is termed thermodynamic coupling.

There is ample evidence to suggest that this coupling can become important in certain circumstances. For instance, Tewfik *et al.* [6] found that the additional heat flux due to coupling was of the same order of magnitude as, and in the opposite direction to, the familiar Fourier flux. A comprehensive theoretical and experimental investigation of flame structure and reaction kinetics by Dixon-Lewis *et al.* [3, 4] shows that in a fuel-rich hydrogen–oxygen–nitrogen flame the hydrogen flux due to thermal diffusion may be as large as the ordinary diffusional flux.

Coupling can also be important in multicomponent jet flows which involve large temperature gradients. Thomann and Baron [1] have presented an experimental investigation of thermal diffusion effects in laminar and turbulent shear flows. They used a helium–nitrogen mixture, initially at the same concentration in the jet and the stagnant ambient gas, with a temperature ratio between the jet and its surroundings of about 4 (the jet was at 78°K whilst the ambient gas mixture was at room temperature). The results of the experiment indicate a decrease in the helium concentration in the laminar jet due to thermal diffusion. Under steady-state conditions the concentration was as much as 7% lower than in the surroundings. A theoretical investigation of this problem was carried out by Chien [2] who obtained solutions by iterative integration of the coupled concentration and energy equations. Reasonable agreement with the available experimental data was produced. However, Chien's solution procedure is restricted solely to a binary mixture, and cannot be readily extended to deal with a larger number of species.

In what follows, a general solution procedure is presented for a *multicomponent* axisymmetric laminar jet. A method of solving for the concentration, temperature and velocity fields is suggested. Its application to the particular case of the above mentioned [1] helium–nitrogen jet shows (a) its efficacy as a solution procedure, (b) the important role that thermal-diffusion can play in multicomponent jets of this sort.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Consider an axisymmetric laminar jet of a nonreacting multicomponent gaseous mixture issuing into still surroundings (Fig. 1). In the steady state the governing equations are:

$$\text{continuity:}\ \frac{\partial}{\partial x}(\rho u) + \frac{1}{y}\frac{\partial}{\partial y}(y\rho v) = 0, \tag{1}$$

$$\text{momentum:}\ \rho\left\{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right\} = \frac{1}{y}\frac{\partial}{\partial y}\left(y\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\text{energy:}\ \rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = -\frac{1}{y}\frac{\partial}{\partial y}(yJ_{qy}) + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$\text{species:}\ \rho u\frac{\partial c_i}{\partial x} + \rho v\frac{\partial c_i}{\partial y} = -\frac{1}{y}\frac{\partial}{\partial y}(yJ_{iy}) \tag{4}$$

$$[i = 1, 2, \ldots (n-1)].$$

The nth species concentration can be found using the identity:

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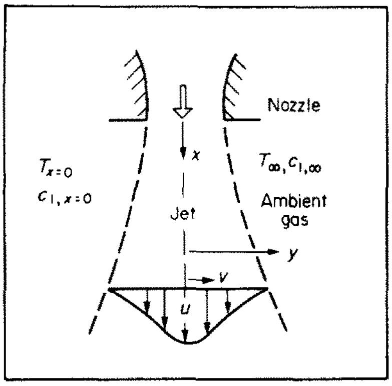

FIG. 1. Axisymmetric jet: axes and boundary conditions.

$$\sum_{i=1}^{n} c_i = 1. \tag{5}$$

In problems of the type in which large differences in temperature and/or concentration between the jet and its surroundings are present, only the terms resulting from diffusion, conduction and thermodynamic coupling in the radial direction are considered. The corresponding axial terms may be assumed negligible.

The thermodynamically coupled mass and energy fluxes in the radial direction are given by:

$$J_{iy} = -\sum_{j=1}^{n-1} D_{ij}\frac{\partial c_j}{\partial y} - D_{iT}\frac{\partial T}{\partial y}, \tag{6}$$

$$J_{qy} = -k\frac{\partial T}{\partial y} - \sum_{i=1}^{n-1} D_{Ti}\frac{\partial c_i}{\partial y} + \sum_{i=1}^{n-1} h_i J_{iy}. \tag{7}$$

If the surroundings into which the jet issues are at a temperature T_∞ the boundary conditions that complete the formal definition of the problem are:

$$y = 0: v = 0, \frac{\partial u}{\partial y} = 0, \tag{8}$$

$$y \to \infty, u = v = 0. \tag{9}$$

For $x = 0$, $|y| \le r_0$ $c_i = c_i(0)$, $T = T(0)$, (10)

$|y| > r_0$ $c_i = c_i(\infty)$, $T = T(\infty)$, (11)

where r_0 is the radius of the jet at $x = 0$.

$$\text{For } x > 0, \quad y = 0, \frac{\partial c_i}{\partial y} = \frac{\partial T}{\partial y} = 0, \tag{12}$$

$$y \to \infty, \quad c_i = c_i(\infty), \quad T = T(\infty). \tag{13}$$

THE ENERGY-SPECIES TRANSFORMED FIELD AND SOLUTION PROCEDURE

Solution of the problem above defined is considerably hindered by the strong thermodynamic-coupling terms. In order to facilitate a solution, a special decoupling transformation (see [8] for details) is applied to equations (3) and (4), leading to a set of uniform 'energy-species' equations having the form:

$$\rho u\frac{\partial \zeta_i}{\partial x} + \rho v\frac{\partial \zeta_i}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left[y\frac{\mu}{\tilde{P}r_i}\frac{\partial \zeta_i}{\partial y}\right] + \frac{1}{y}\frac{\partial}{\partial y}\left[y\mu\left(1 - \frac{1}{\tilde{P}r_i}\right)\frac{\partial}{\partial y}\left(\frac{1}{2}u^{*2}\right)\right] \tag{14}$$

$(i = 1, 2, 3, \dots n)$.

Here ζ_i is the transformed 'energy-species' variable defined by:

$$\zeta_i = \sum_{j=1}^{n-1} \alpha_{ij} c_j + \beta_i I^* \tag{15}$$

where α_{ij} and β_i are coefficients resulting from diagonalization of the phenomenological coefficients matrix, and the equivalent 'Prandtl–Schmidt' number $\tilde{P}r_i$ is related to the matrix's eigenvalues.

Thus, if the values of ζ_i are determinable from equation (14), the concentration and enthalpy fields can be reclaimed via the reverse transformation:

$$(c_i, I^*) = \left(\sum_{j=1}^{n} \gamma_{ij}\zeta_j, \ \sum_{j=1}^{n} \gamma_j\zeta_j\right). \tag{16}$$

The continuity and momentum equations remain unchanged. The boundary conditions for the ζ_i can be readily deduced by similar application of the transformation to (10)–(13).

The form of the transformed governing equations makes them particularly amenable to solutions by the well-documented method of Patankar and Spalding [9, 10] for parabolic problems (the method does not appear to have been utilised for problems in which thermodynamic coupling is present). Equations (14) are exactly similar to the stagnation enthalpy equation in the absence of coupling. Hence, in the GENMIX program [9, 10] the usual solution for enthalpy and species is simply replaced by that for n transformed variables. After each step of integration downstream, the species and temperature profiles are recovered using equation (16).

This procedure is *general* in nature, and circumvents solution attainment difficulties engendered by the coupling terms.

CALCULATED RESULTS

To illustrate the above-described procedure, and the effects of thermodynamic coupling, thermal diffusion in a laminar jet composed of helium and nitrogen was examined. The data for the various transport coefficients were culled from a number of sources [2, 12–15]. The general qualitative behaviour of the concentration profiles calculated was in agreement with the results of Chien [2] and the experimental data [1] (see Fig. 2), although slightly higher values of thermal separation were predicted here (possibly due to local constancy approximations inherent in applying the decoupling transformation). Maximum separation due to thermodynamic coupling occurs for an average concentration of about 0.55 of helium.

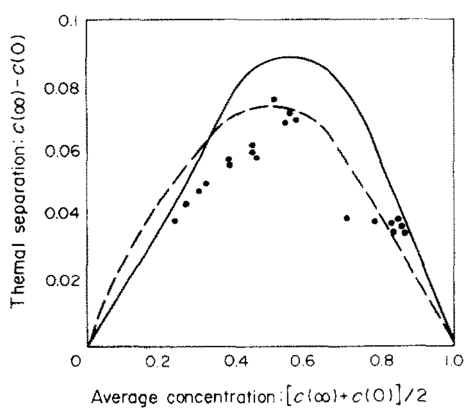

FIG. 2. Effect of thermal diffusion; ——— present theory, ----- Chien's [2] solution, ... experimental results [1].

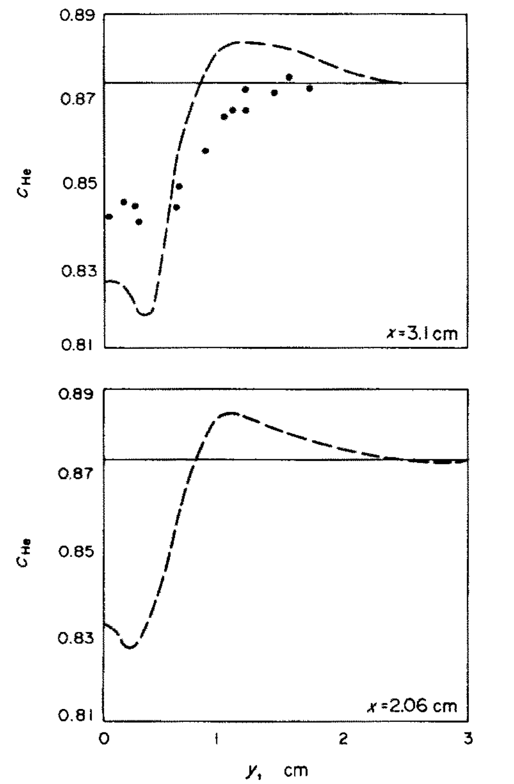

FIG. 3. Development of helium concentration profile with axial distance; ----- present theory, ——— without coupling, ... experimental results [1].

Development of the concentration profile with axial distance is illustrated in Fig. 3. The inlet concentration of helium is 0.8735, corresponding to the *only* case for which a radial profile is presented in the experimental data [1]. The local over- and undershoots (of about 1.3 and 6.6% respectively)

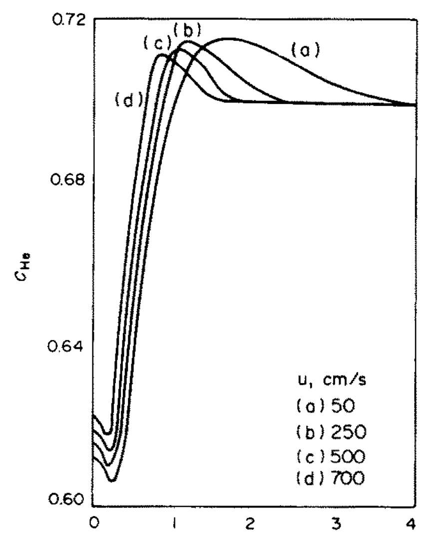

FIG. 4. Effect of the jet velocity on helium concentration distribution.

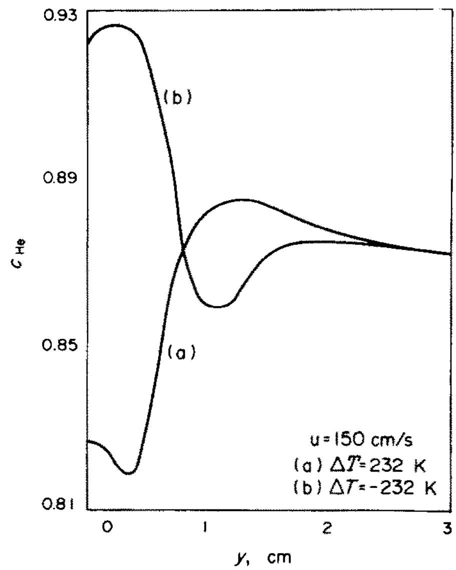

FIG. 5. Helium concentration profiles for reversed temperature differentials: $\Delta T = T(\infty) - T(0)$.

can only be attributed to thermal diffusion, as evidenced by comparison with the profiles obtained with the coupling terms neglected. In the latter case the initial concentration profile remains basically unchanged. However, it should be added, in contrast, that the temperature profiles computed with and without coupling were virtually identical. At the station $x = 3.1$ cm where there are the only measurements of the radial profile, the qualitative agreement with experiment is fair, although the overshoot in the helium concentration was not detected experimentally (see also [2]).

For a given value of the He concentration at the jet exit, the effect of thermal diffusion is reduced, at a fixed downstream x-station, as the jet inlet velocity is increased (Fig. 4). Although the effect is not large, the predominance of the convective term begins to outweigh the thermal diffusion for higher velocities.

Finally, for a given jet velocity and ambient He concentration, the effect of reversing the temperature differential was examined (Fig. 5). For the hotter jet the migration of the helium is in the direction of the jet axis, causing a subsequent overshoot of about 6%. The resulting depletion of helium near the colder boundary is of the order of 1.9%. These results indicate the extreme sensitivity of thermal diffusion to the different conditions.

CONCLUSIONS

The applicability of the solution procedure described above has been illustrated for a binary mixture; the method, however, is *general* and can be used for jets comprised of many species.

For the sorts of operating conditions examined here, thermodynamic coupling cannot be neglected *a priori*, particularly if there are lighter species present in the multicomponent jet. The present general approach permits the determination of the range of conditions under which the coupling effects are significant to be analysed with relative ease.

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COMBINED FREE AND FORCED CONVECTION NEAR A VERTICAL WALL DUE TO OSCILLATIONS IN THE WALL VELOCITY AND TEMPERATURE

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NOMENCLATURE

g, acceleration due to gravity;
β, coefficient of volume expansion;
α, thermal diffusivity;
ν, coefficient of kinematic viscosity;
$\bar{T}_w, \bar{T}_\infty$, constant mean wall temperature, constant free stream temperature;
U_0, mean velocity of wall oscillations;
ε, a very small number;
δ, dimensionless boundary-layer thickness;
Pr, Prandtl number;
x, y, coordinates in x, y directions;
η, non-dimensional perpendicular distance from the wall;
U, V, T, t, ω, fluid velocities in x, y directions, fluid temperature, time, frequency of oscillations in their non-dimensional forms. Barred quantities denote their dimensional forms.

Subscripts

s, steady part;
1, 2, oscillating component, out of phase component;
x, y, t, partial differentiation with respect to these variables.

1. INTRODUCTION

Study of combined free and forced convection near a wall whose temperature and velocity oscillate about a mean is important from a practical point of view. Uniform velocity or constant wall temperatures are only ideal cases and in reality are subject to periodic variations occurring at long intervals which is a case of low frequency oscillations or at short intervals corresponding to high frequency oscillations. The variations may not be strictly periodic but may very nearly be so. The vertical motion of a rocket through still air having such approximate small periodic changes in its velocity and wall temperature can be likened to a model of flat plate in motion with small variations (from their constant values) in its velocity and temperature. Study of high frequency oscillations in heat transfer near a wall which might be in periodically varying relative motion is of some consequence in the working of liquid rocket and turbojet engines.

Nanda and Sharma [1] studied the free convection in the boundary layer near a vertical wall with its temperature oscillating about a non-zero mean. We have extended their study by imposing a motion on the flat wall varying periodically about a steady mean. Though the manner of analysis follows the lines of Nanda and Sharma [1], some new results evidently reflecting the effects of the imposed oscillatory motion have been obtained and are expected to be of practical interest in problems of the nature quoted above. It is of special interest to note that in case of high frequency oscillations there always exists a fluid layer parallel to the wall at a distance $\eta = 0.37$ which travels with a velocity oscillating

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